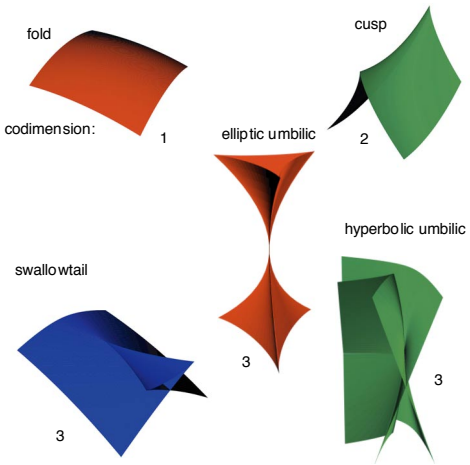
CATASTROPHE OPTICS

Catastrophe optics is about rainbows, the sparkling of the sun on the sea, the dancing focal lines painted by the sun on the sand beneath shallow water, twinkling starlight... It is the optics of nature rather than artefact. This branch of the physics of light has developed over the last three decades under the stimulus of a remarkable mathematical theory by Thom and Arnold, which forms the basis of 'catastrophe theory'.

Catastrophe optics is about the focusing of rays, and associated wave interference and diffraction phenomena, in the absence of symmetry. It differs from conventional studies of focusing, which concentrate on perfect symmetry and treat small departures therefrom as 'aberrations'.

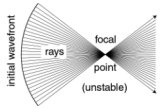
The mathematical theory classifies ways in which critical points (maxima, minima, saddles) can coalesce as parameters are altered. It applies to light because of Fermat's principle: light rays are critical points of an optical distance function. Focusing occurs on the envelopes where neighbouring rays touch, and so is precisely the coalescence of critical points. The envelopes are curves in the plane, or surfaces in space, called caustics. The mathematics classifies 'generic' or 'structurally stable' caustics, whose form is unaffected by small disturbances.

In catastrophe theory, caustics are classified by their codimension, that is by the number of parameters needed to specify them. Thus the fold, with codimension 1, can appear as a point on a line, a smooth curve in the plane, or a surface in space. Here is the complete list of catastrophes up to codimension 3:



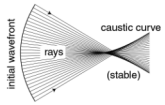
Note that this list is exhaustive. The remarkable mathematical achievement was to show that in three-dimensional space there can be no stable caustics other than these

This is the ideal point focus of a perfect lens or mirror



It is not structurally stable because almost any perturbation causes the point to explode ('unfold') into a cusped caustic in the plane:

And indeed the cusp is one of the 'elementary catastrophes' on the mathematicians' list. Such a caustic is a familiar sight whenever light from a compact source is reflected in a teacup.



Each brilliant point is a specular image of the sun in the water, and corresponds to a critical point in the distance sun-water-eye. Over time, the water surface changes as the waves move, and these points are born or annihilate in pairs. At such a 'twinkle' the water surface is not only oriented to reflect light into the eye but is curved so as to focus it there. This interaction of two critical points, as a single parameter (time) varies, is the simplest catastrophe, called the fold. Another familiar fold is the rainbow, where the single parameter is the deflection angle of sunrays by raindrops.

The sun is low in the sky and the sea is calm. Like a mirror as they say. Only it is not like a mirror. The waves which are scarcely waves, for they come and go in many different directions and their rising and falling is barely perceptible. are made up of innumerable tiny surfaces at variegating angles to one another - of these surfaces those which reflect the sunlight straight into one's eyes, sparkle with a white light during the instant before their angle, relative to oneself and the sun,shifts and they merge again into the blackish blue of the rest of the sea. Each time the light lasts for no longer than a spark stays bright when shot out from a fire. But as the sea recedes towards the sun, the number of sparkling surfaces indeed looks somewhat like a silver mirror. But unlike a mirror it is not still. Its granular surface is in continual agitation. The further away the ricochetting grains, of which the mass become silver and the visibly distinct minority a dark leaden colour, the greater is their apparent speed. Uninterruptedly receding towards the sun, the transmission of its reflections becoming ever faster, the sea neither requires nor recognizes any limit. The horizon is the straight bottom edge of a curtain arbitrarily and suddenly lowered upon a performance. (Last paragraph of the novel 'G', by John Berger)

The details of these swimming-pool caustic networks are obscured by their rapid motion and by the finite disk of the sun. But they must obey the rules of catastrophe theory, and close examination reveals fine structure.