PHYSICS-MATHEMATICS LEXICON

OPTICS	CATASTROPHE THEORY
Coordinates **S** on an initial wavefront	State variables **S**=$(S_1 ... S_n)$
Coordinates **C** describing position and time in wavefield	Control parameters **C**=$(C_1, C_2 \ldots)$
Optical distance from **S** to **C**	Potential function $f(\mathbf{S}, \mathbf{C})$
Fermat's principle gives coordinate(s) **S** from which ray(s) travel to event at **C**	Gradient map $\nabla_{\mathbf{S}} f(\mathbf{S}, \mathbf{C})=0$ gives critical points in state space
Caustic (focal manifold) in **C**, i.e. envelope of rays	Locus of **C** for which f has degenerate critical points in **S**
Moving onto a caustic in **C**	Changing parameters **C** so that critical points coincide in **S**
Structurally stable caustic	Singularity of gradient map, i.e. elementary catastrophe in **C**

Standard polynomials $f(\mathbf{S},\mathbf{C})$

fold	$f = S^3 + CS$
cusp	$f = S^4 + C_2 S^2 + C_1 S$
swallowtail	$f = S^5 + C_3 S^3 + C_2 S^2 + C_1 S$
elliptic umbilic	$f = S_1^3 - 3S_1 S_2^2 + C_3\left(S_1^2 + S_2^2\right) + C_1 S_1 + C_2 S_2$
hyperbolic umbilic	$f = S_1^3 + S_2^3 + C_3 S_1 S_2 + C_1 S_1 + C_2 S_2$

Intensity $I(\mathbf{C})$ for diffraction catastrophe with wavelength λ:

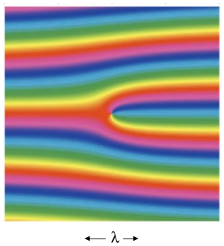
$$I(\mathbf{C}) \propto \left| \frac{1}{\lambda^{n/2}} \int \mathrm{d}\mathbf{S} \exp\left\{ \frac{2\pi \mathrm{i}}{\lambda} f(\mathbf{S},\mathbf{C}) \right\} \right|^2$$

At the most singular point $(\mathbf{C} = 0)$, $I \propto \lambda^{-\mu}$, where μ is the 'singularity index':
$\mu = \frac{1}{3}$ (fold), $\mu = \frac{1}{2}$ (cusp), $\mu = \frac{3}{5}$ (swallowtail),
$\mu = \frac{2}{3}$ (elliptic and hyperbolic umbilics)

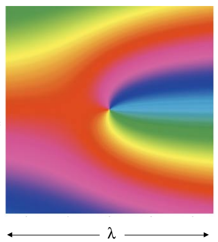
PHASE SINGULARITIES - MUCH ADO ABOUT NOTHING

On caustics, waves reach their greatest intensity. At the opposite extreme - and indeed complementary to caustics - are places where the wave intensity is zero. These are points in the plane, or lines in three dimensions. They are singularities of the phase of the wave, where the phase takes all values. Since phase is a periodic variable (0 and 2π are equivalent), it is natural to represent it by colour, with hues progressing from red through yellow green, blue and purple back to red:

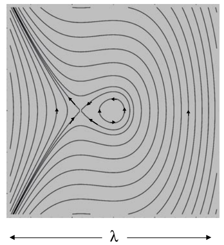
Here is a plane wave with a phase singularity; the resemblance to a crystal with an extra half-plane of atoms has led to phase singularities being called wavefront dislocations; the analogy is far-reaching:



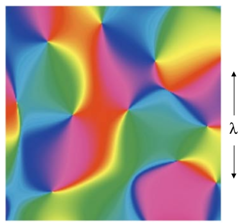
Zooming in reveals the structure of the phase singularity (distances are in units of wavelength):



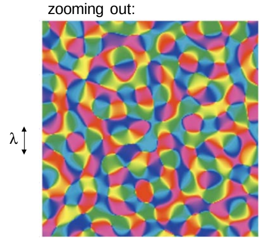
Plotting the lines of energy flow (current vector), shows that the dislocation is a vortex in the associated current, and that there is a phase saddle only 1/2π wavelengths away). Many experiments now study these 'optical vortices' in laser beams.



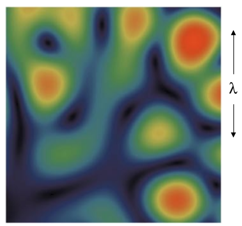
Random waves, such as this superposition of plane waves, contain many phase singularities. They are delicate features, and in nonmonochromatic waves can move faster than light- -without contradiction, since they carry no information or energy

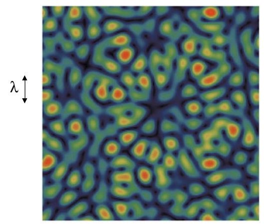


zooming out:



The corresponding intensity maps show a rich structure of correlations, whose statistics are being studied now. These random waves are models for the fine-structure of black-body radiation, and for wavefunctions in 'quantum chaology', where the corresponding classical motion (for example of an electron in 'quantum dot' threaded by a magnetic field) is chaotic.





Phase singularities were discovered by William Whewell in 1833, in the wavefronts of the tides. These are the 'co-tidal lines' linking places where the tide is high at a given time. Dislocations are 'amphidromies', around which the wavefronts rotate like the spokes of a wheel. Here is an amphidromy in the North Sea:

Whewell