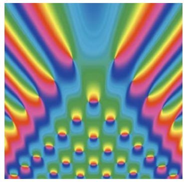
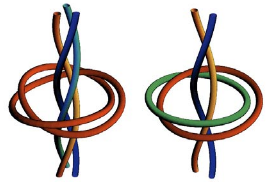
The complementarity between phase singularities and caustics can be seen in phase maps of the diffraction catastrophes, as shown here for the cusp. The dislocations are fine details, forming an array inside the geometrical cusp and a row outside the cusp on each side

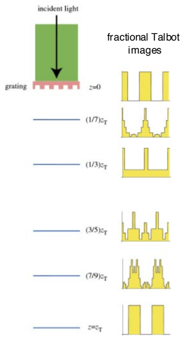


In three dimensions, dislocation lines can form complicated space curves. Outside the elliptic umbilic diffraction catastrophe, for example, there are dislocations in the shape of curly antelope horns. Or, the dislocations can form knots and links, threaded by multistranded helices, as here. These knotted zeros can be produced by interfering laser beams, or by appropriate superpositions of the degenerate quantum states of hydrogen.

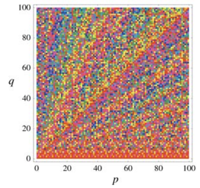


CARPETS OF LIGHT, QUANTUM CARPETS

Henry Fox Talbot is known as one of the inventors of photography, but he also made many discoveries in optics. In 1836 he noticed, while inspecting the field behind a diffraction grating with a magnifying lens, that the grating came into sharp focus not only close to the lens but also at multiples of a distance $z_T=a^2/\lambda$, where a is the spacing of the slits of the grating and λ the wavelength of the light. More complicated images form at rational multiples $(p/q)z_T$, where p and q are integers. Thus the mathematics of number theory appears in the simplest problem of diffraction physics. This is an intricate interference phenomenon.

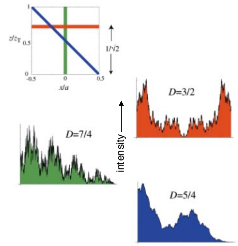


The 'fractional Talbot images' are coherent superpositions of the images of the slits of the grating. The phases of the images are given by beautiful sums discovered by Gauss at the end of the 18th century. Each phase is the direction of the resultant when q unit vectors are added in the (complex) plane, with the angle between successive pairs of vectors increasing by $\pi p/q$, starting with a separation $2\pi n/q$ for the nth image. Underlying the complexity of the Talbot images is the irreducible complexity of the Gauss sums:

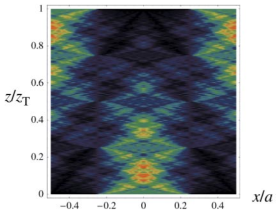


phases of Gauss sums, colour-coded

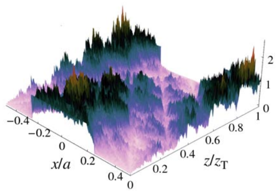
Behind a grating with sharp slits, the field has a fractal structure of fantastic complexity. Between the Talbot images, that is for irrational fractions of z_T, the graph of intensity across an image is a curve with fractal dimension 3/2. Longitudinally, that is, parallel to the incident light, the intensity graph is a fractal with dimension 7/4. And there are infinitely many diagonal directions where the intensity fractal has dimension 5/4. This fractality extends over a range of scales from a to λ.



These fractal structures can be visualized in density plots ('carpets') of the Talbot intensity :

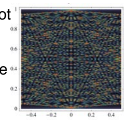


...or as a 3D plot of the 'Mountains of Talbot:

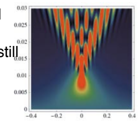


For a smooth (e.g. sinusoidal) phase grating, the geometrical rays would form cusped caustics:

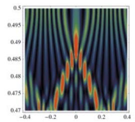
But according to the Talbot effect the field should be longitudinally as well as transversely periodic. here are two periods of this 'Talbot carpet'



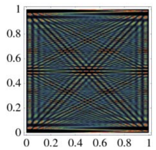
Nevertheless, the geometrical cusps - decorated with their diffraction catastrophes - are still present, as this (anisotropic) magnification reveals:



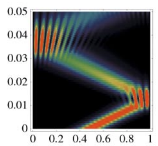
The fractional Talbot effect implies that phantom cusps are miraculously created by interference at intermediate distances. Here is a cusp Talbot-reconstructed at $z_T/2$:



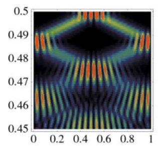
The Talbot-effect mathematics, but with time replacing propagation distance, describes the motion of quantum wavepackets, for example electrons in distant orbits (highly excited states) in a hydrogen atom, or, as in this spacetime density plot ('quantum carpet') for a particle in a box:



Magnification shows the initially Gaussian wavepacket bouncing off the walls of the box while spreading:



However, after a certain time (the analogue of the Talbot distance), the wavepacket is recreated by interference in a 'quantum revival'; here is a double fractional revival at half of this time:



For more about physical asymptotics, see http://www.phy.bris.ac.uk/staff/berry_mv.html

This section is based onthe article 'Quantum carpets, carpets of light', by Michael Berry, Irene Marzoli and Wolfgang Schleich, Physics World 2001