Book reviews

Physics

The optics of rays, wavefronts and caustics

By O. N. STAVROUDIS. *pp. 314. New York and London: Academic Press, 1972. £8.75.*

This is a book about geometrical optics, with the particular aim of providing 'a concrete link with the underlying mathematical foundation'. The range of physical phenomena covered by this treatment is narrow, the principal omission being crystal optics. This is a pity, because it would not have been difficult to extend the theory to cover propagation in anisotropic materials, and to include polarization effects. Another omission is a systematic discussion of the Hamiltonian theory of material systems; it is true that much of the mathematics would also apply to, for example, electron optics, but this is not emphasized.

However, within these limitations of physical content, the author's treatment is splendidly original. Indeed, this is not only a treatise on geometrical optics, but also a very readable exposition of large tracts of 'applicable mathematics', in which the development of each topic is linked with the introduction of a new physical concept.

The first such topic (after a useful historical introduction) is the *calculus of variations*; this formalism is necessary, because the whole work is founded on Fermat's principle that ray paths travel between two points in the shortest (or longest) time. Inhomogeneous media are included in the treatment, which is general enough to deal properly with the tricky case of a sudden change in refractive index, e.g. at the surface of a lens.

The need to describe ray paths leads into *differential geometry of space curves*, including the Frenet formulae, curvature, torsion, etc. *Vector notation* comes in naturally here, and is used throughout the book.

Wavefronts are introduced as surfaces orthogonal to families of rays. To describe them, the author gives a detailed account of the *differential geometry of surfaces* described by two arbitrary parameters. The notions of normal section and principal curvatures are introduced, the important theorems of Meusner and Gauss are proved, and the Weingarten equations (for the change in direction of the normal as a surface is traversed) are derived

To show how the propagation of wavefronts can be studied without first finding the orthogonal system of rays, the eikonal equation is derived. To discuss it, a description is given of the theory of *nonlinear partial differential equations*. This is introduced via a particularly lucid discussion of *total differential equations* and the condition for their integrability, which is also required in deciding whether a given family of rays possesses a set of wavefronts.

The last major branch of mathematics developed in this book is *group theory*. This occurs in the last chapter, where it is shown that the action of a series of lenses on a family of rays, or on a wavefront can be considered as the application of a composition law to the elements of a group—the elements are the lenses themselves, and composition is the operation of placing them together. In this context we are introduced to the linear group, Lie groups, subgroups, symplectic groups, generators, etc.

In practical optical design the important mathematical problem is, fairly obviously, the tracing of rays and wavefronts through a system of lenses, and this is one of the main themes of the present work. It is explained how the basic processes of *refraction* at surfaces of discontinuity of the refractive index, and *transfer* between such surfaces, can be treated separately. The analysis is presented first for rays, and then for wavefronts. Finally, it is shown how the whole process is conveniently described using matrix notation: successive multiplication by refraction and transfer matrices gives the 'image' wavefront resulting from a given 'object' wavefront.

An elegant treatment of the complicated theory of aberrations is given in terms of a power series expansion of the eikonal function (this function gives the optical path length from any given point). A simple example is given which demonstrates that this series sometimes fails to converge, but the implications of this are not discussed—perhaps the general 'aberration series' is asymptotic, so that the validity of conventional analyses, which invariably restrict themselves to the first few terms, is not imperilled.

One very pleasant feature of this book is a discussion of *caustic surfaces*; these are the envelopes of families of rays. As the author says: 'The caustic is one of the few things in geometrical optics that has any physical reality. Wavefronts and rays are not realisable; they are just convenient symbols on which we can hang our ideas. The caustic, on the other hand, is real and becomes visible by blowing a cloud of smoke in the region of the focus of a lens'. How true this is, and how little appreciated! The author proves that in a uniform medium the caustics are loci of centres of curvature of any wavefront, but does not indicate whether any related construction can be employed in an inhomogeneous medium. Probably not, but in any case, the structure and propagation of caustics would be a promising new field of research, in mechanics as well as optics. In this context the author's concept of the *archetypical wavefront* would probably be useful; this is a particular wavefront which, if it propagated without refraction, would reproduce the caustics, etc., of an actual ray system that does undergo refraction.

I am not so sure what to make of the derivation, from geometrical optics alone, of what the author cautiously calls the 'pseudo-Maxwell equations'. These have exactly the same form as the general equations for electromagnetic fields in an arbitrary inhomogeneous medium specified by a refractive index n. However, instead of the electric field at a point, we have n times the unit normal vector to a member of a family of rays at that point, and instead of the magnetic field we have n times the unit binormal vector of the ray. The family of rays must of course satisfy the equations of geometrical optics in the medium. The pseudo-Maxwell equations should not, however, be taken to imply that electromagnetism can be derived from geometrical optics alone, for the following reason: the medium is not completely specified by n alone; *two* independent functions are required, namely the dielectric constant and magnetic permeability. But in the author's derivation these are *defined*, in terms of line integrals of the ray curvature. To put it differently, exactly similar arguments based on geometrical acoustics would lead to equations describing *sound* in an inhomogeneous fluid as a *transverse* wave! Nevertheless, the pseudo-Maxwell equations are puzzling, and further research is required to elucidate them.

To illustrate the formalisms developed in the book, several examples are studied in detail: Maxwell's fish eye (an inhomogeneous medium where every point has a perfect image), the heated window (a slab across which the refractive index varies linearly), and refraction across a plane interface (including a discussion of the 'virtual caustics'). However, this is not enough, and the book would have been much better if a large number of worked examples and exercises of varying difficulty had been included. This would have been especially useful in elucidating the mathematics.

The author writes in an easy, witty style which greatly helps in understanding the rather formal subject-matter The standard of production is high (although I noticed half-a-dozen minor mistakes) and the printing is elegant. This book is recommended for the physics libraries of universities and technical laboratories, and to optical designers wishing to understand the inner mathematics of their craft.

MICHAEL BERRY

General

A random walk in science

Compiled by R. L. WEBER. *pp. 206. London: The Institute of Physics, 1973. £4.75.*

This is a well-meant book with some good things in it. It began as a collection of humorous items about physics, made by R. L. Weber, and has been edited, with considerable modifications, by E. Mendoza. The inclusion of such items as an account of the trial of Galileo, some slightly suggestive X-ray topographs by Andrew Lang or an encomium on the experimental genius of Rutherford, has added several more dimensions to the stochastic trajectory of the reader, with the intention of entirely losing him in the clouds. Trying to review such an amorphous and incoherent anthology, I feel a bit disoriented myself.

There are several good jokes. Frisch's gem 'On the feasibility of coal-driven power stations' is a subtle attack on technical expertise. Denys Wilkinson's 'Slidesmanship' is a masterly exposition of one of the minor techniques of oneupmanship by a leading authority. 'A contribution to the mathematical theory of big game hunting' is a succint analysis of mathematical logic in action. And the random sentence generator that Gulliver observed in Laputa is too close to Haiku programming for our comfort. There are also various charming anecdotes: R. W. Wood's exposure of Blondlot's N-Rays; attempts at conversation with Bohr; Jeremiah Horrox's observations on the transit of Venus. For these little items alone, this collection is worth having.

But a lot of jocularity is a bit forced. Without a genuine touch of satire a lengthy parody of technical jargon can be as boring as the follies it apes. A funny story is insipid without a dash of acid wit. Comical exam answers are not so laughable, after all, as the pompous questions that prompt them. Little poems to the tune of Clementine about ions going round magnetic lines were, no doubt, great fun at the Cavendish Xmas Party, but don't survive in cold print. For laughing out loud, give me Groucho Marx, every time, or almost any canonical limerick.

So I wish they had been more selective —and that they had sought farther. What about Pauli saying to Weisskopf 'I should have taken Bethe' . . .? Or Hilbert's funeral oration for the student who nearly proved the theorem about the zeros of the Riemann zeta function? Or Dirac's response 'that wasn't a question; it was a statement'? Or the exploits of Dr Grant Swinger? For, as Oliver Edwards explained to Boswell 'I have tried too in my time to be a philosopher; but I don't