

is, a thermal plasma at a temperature of about  $10^{12}$ K.

Obviously, Faraday rotation measurements are not directly sensitive to protons. But if electrical neutrality is preserved, they do strongly limit the proton number density.

The observations tell us that there are very few non-relativistic electrons inside the radio-emitting region, and, therefore, that the acceleration mechanism must be extremely efficient. This is the fundamental constraint that must be satisfied by whatever acceleration process is invoked.

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### Topography of random surfaces

SAYLES and Thomas have claimed that the topographies of natural and artificial random surfaces commonly possess isotropic spectra  $G(\omega)$  which fall with spatial frequency  $\omega$  according to the power law

$$G(\omega) = 2\pi k / \omega^2 \quad (1)$$

where  $k$  (which they call the 'topothesis') is a constant depending on the surface chosen, and  $\alpha$  takes a universal value of 2.

They derive  $\alpha=2$  from a theoretical argument based on the assumption that above any horizontal straight line the changes of surface height  $\Delta h_{12}$  and  $\Delta h_{23}$  along consecutive segments 1, 2 and 2, 3 of the line are statistically independent. But there is no reason *a priori* that this assumption should hold in practice. For a general  $\alpha$  there will be correlation between the height change, even for line segments separated by an arbitrarily large distance. This is consistent with their principle that "a sample of finite length taken from such a surface will never, however long, completely represent its properties". Statistically isotropic surfaces which have no scale and whose height function is well defined but non-differentiable may have spectra of the form of equation (1) with any  $\alpha$  in the range  $1 < \alpha \leq 3$ . Their geometric properties are discussed in detail by Mandelbrot<sup>2</sup> who calls them 'fractals' to emphasise that they can be considered as having fractional dimensionality

$$D = (7 - \alpha) / 2 \quad (2)$$

lying between 2 and 3. Sayles' and Thomas' argument is restricted to the 'Brownian' case  $D=2.5$ . In the general case their useful concept of topothesis is better defined as the length

$$T = k^{1/(3-\alpha)} \quad (3)$$

which is proportional to the horizontal distance,  $L$ , between two points on the surface whose connecting line has r.m.s. slope unity,  $(J\langle(\Delta h)^2\rangle)^{1/2} = L$ .

Experimental evidence is adduced to support  $\alpha=2$  but the treatment of the data is procrustean. Their Fig. 2 combines measurements on 23 types of surface spanning nearly eight decades in wavelength. However, the points representing the individual types of surface cover much smaller ranges and inspection shows that when fitted with spectra of the form (1) their separate  $\alpha$  vary from 1.07 to 3.03 (nicely covering the fractal range). Sayles and Thomas seem to have fitted each set of data to a prescribed spectrum with  $\alpha=2$  thus determining the constant  $k$ , and then plot all the 'reduced' spectra  $G(\omega)/k$  on the same graph. On a plot with axes  $x = \log(2\pi/\omega)$ ,  $y = \log G$  this amounts to taking raw data consisting of 23 short line segments with varying slopes scattered over the  $x, y$  plane and translating each one along  $y$  so that it lies as close as possible to the line  $y=2x$ , a procedure guaranteed to produce a better looking fit as the total range covered by the data increases.

Therefore the claim to have discovered a "common natural law underlying the phenomena" of surface topography cannot be justified or supported by the experiments they cite.

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SAYLES AND THOMAS REPLY—We welcome the comments of M. V. Berry and J. H. Hannay<sup>1</sup> and would like to take the opportunity to clarify some of their points. We feel that their objection to our derivation is a misunderstanding caused by our invoking the theory of Wiener<sup>2</sup>, as we do not require the changes in height to be statistically independent, only the heights themselves. An unambiguous outline of our derivation is discussed in the first paragraph of our letter.

Our Fig. 2 was, in fact, constructed by the technique Berry and Hannay suggest but this was through necessity and was not intended to appear procrustean. We could find no better and unambiguous technique for comparing the experimental points from the 23 different sources with our derived model. As most surface measurement techniques can only record information over approximately two decades of wavelength, we would expect a variation in  $\alpha$  of the  $\omega^{-\alpha}$  (where  $\alpha=2$ )

law to occur. Although we have no formal proof we consider our derived  $\alpha=2$  as the mean condition and the variation of  $1 < \alpha < 3$  suggested by Mandelbrot<sup>3</sup> as being an upper and lower limit of possible variation. This implies that the probability of obtaining  $\alpha \neq 2$  increases as the bandwidth reduces, which appears to be the case with our data. The longest spectrum we show, a supertanker hull plate, covers approximately four decades and although  $\omega^{-2}$  is not the best fit within short segments, over the total range it does represent the best  $\omega^{-\alpha}$  power law fit. Our Fig. 2 depicts all of what one might call the worst experimental points, many points which would fit an  $\omega^{-2}$  law more closely had to be omitted to avoid unnecessary confusion.

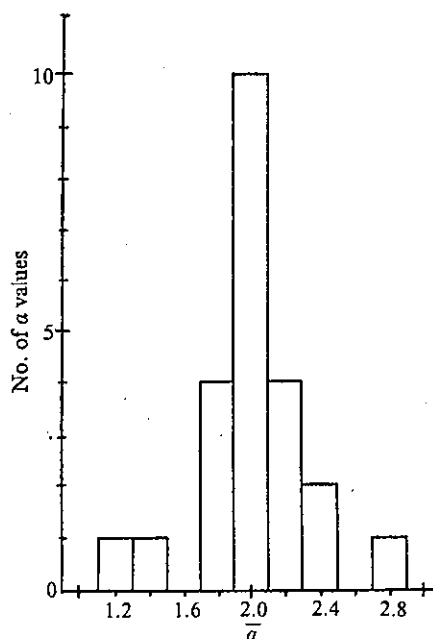


Fig. 1 Histogram of the values of  $\alpha$  obtained from fitting a power law of the form  $\omega^{-\alpha}$  to the 23 sets of experimental points of our previous Fig. 2 (ref. 1).

Without adding the points omitted from Fig. 2 we have replotted all the spectra as Berry and Hannay did, and the values of  $\alpha$  are shown as a histogram in Fig. 1. Mandelbrot (personal communication) suggests that  $\alpha$  should take a value of about 2.5, whereas we find from Fig. 1 (as Berry and Hannay must have done, although they failed to mention it), the mean value of  $\alpha$  is 1.98 and, therefore, find no reason to change our claims.

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